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LIFTING SURFACE IN AN UNSTEADY FLOW NEAR A SCREEN (NESUSHCHAYA --ETC(U)
AUG 76 B N BELOUSOV, A N LUKASHENKO

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LIFTING SURFACE IN AN UNSTEADY FLOW NEAR A SCREEN

[Belousov, B. N., A. N. Lukashenko, and A. N. Panchenkov, Nesushchaya poverkhnost' v nestatsionarnom potoke vblizi ekrana, in: Aircraft Construction and Technology of the Air Fleet (Samoletostroyeniye i Tekhnika Vozdushnogo Flota), No. 18, Khar'kov State University Publishing House, Khar'kov, 1970, pp. 3-11; Russian]

Several major works dealing with the theory of a lifting surface in an unsteady flow with various approximate methods of wing design are now in existence.[1,2,3] It is common knowledge that the problem of a thin lifting surface of arbitrary aspect ratio is reduced to two-dimensional integral equations for which there are no closed solutions. /3*

One of the important aspects is the study of two-dimensional integral equations using one-dimensional approximations, and obtaining final results for the hydroaerodynamic characteristics of a wing in an unsteady flow on the basis of these equations.

While the Prandtl lifting line theory defines the Prandtl equation as a one-dimensional approximation for a wing in a steady flow, the complexity of the physical phenomenon gives no unique approximation method for a wing in an unsteady flow. This accounts for the large number of treatments (up to 20, noted by Bisplinghoff, Ashley, and Halfman[1]) which yield various one-dimensional equations.

Panchenkov[6,7,8,9] has developed a method of integral operators of the theory of a lifting surface in an unsteady flow.

The general solution of the problem is in the form of three components, two of which are obtained from equations of the same form as those of the theory of a wing in a steady flow.

Reference 8 also discusses a method of formulating the Prandtl equation by using the example of a wing in an unbounded fluid flow. A considerable advantage of this method consists in the fact that the Prandtl equation was constructed for the parametric constant of the singular solution. This permitted a deeper analysis of the problem and a critical evaluation of the existing theories of Reisner,[1] Küssner,[2] and others.

In the present paper, the method of integral operators is applied to the problem of a lifting surface near a screen. Two-dimensional integral equations and a one-dimensional integrodifferential equation for the parametric constant were obtained. It should be noted that the downwash is related only to the singular solution corresponding to the singularity in the entering wing edge. Transposition of this result to the steady motion theory shows that the classical lifting line theory is strictly applicable only to a plane lifting surface.

If the lifting surface is not planar, terms of order $\frac{1}{\lambda}$, not considered in the classical theory, appear in the expression for the wing lift coefficient. From a physical standpoint, they determine the effect of the aspect ratio on the zero-lift angle of the wing.

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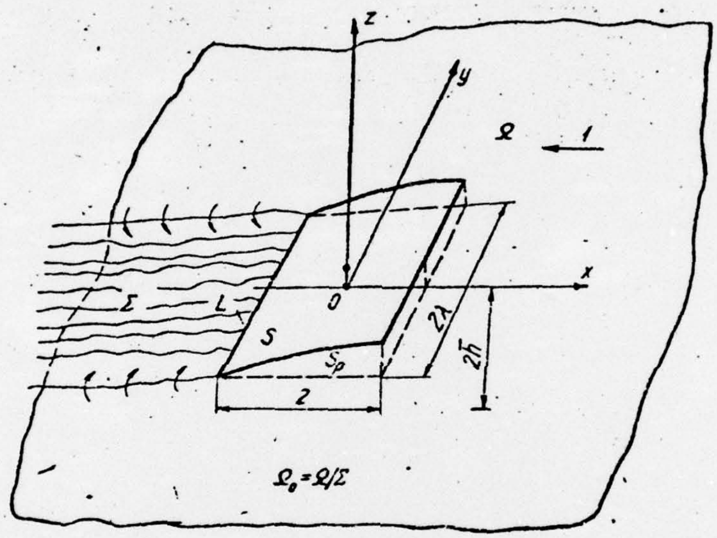
*Numbers in the right margin indicate pagination in the original text.

The present study is closely related to those of Refs. 6, 7, 8, and 9, and we therefore attempted to mention their general results very briefly where necessary, while discussing the new results more extensively.

1. Let a thin lifting surface S move at velocity $V_0$ near a solid screen. In addition to the basic translational motion, it also executes harmonic oscillations of frequency $\omega$. /4

Assuming that the disturbances introduced into the flow by the lifting surface are small, the problem can be linearized in a known manner[4,5] in the coordinate system oxyz moving at velocity $V_0$ (see figure).




In the case of harmonic oscillations, for the hydrodynamic potentials of velocities $\phi$ and accelerations $\Theta$ and their derivatives, we have

$$\Theta^n_{x_i}(g;\ t) = \bar{\Theta}^n_{x_i}(g)\, e^{i\omega t},$$
$$\varphi^n_{x_i}(g;\ t) = \bar{\varphi}^n_{x_i}(g)\, e^{i\omega t}\ g \in \Omega_0. \qquad (1.1)$$

The index n denotes an nth order derivative of the functions $\Theta$ and $\phi$ with respect to the $x_1$ coordinate ($x_1 = x$; $x_2 = y$; $x_3 = z$).

Only the real parts of the corresponding expressions are considered above in Eqs. (1.1), but for simplicity, the symbol Re is omitted. Relations (1.1) make it possible to formulate the problem for $\bar{\Theta}(g)$ and $\bar{\phi}(g)$ and to exclude time from consideration.

Changing to dimensionless space $\Omega$ at an incident flow velocity of 1, and removing the upper dash from the corresponding symbols $\Theta(g)$ and $\phi(g)$, we have a boundary value problem in the space of acceleration potential $\Theta$:

$$\Delta\Theta = 0;\quad g \in \Omega;$$
$$\Theta_z = F(g);\quad g \in S_p; \qquad (1.2)$$

(A)

$$\Theta_z = 0;\quad z = 2\bar{h};$$
$$\Theta_+ - \Theta_- = 0,\quad g \in L;$$
$$\Theta \to 0; \qquad (1.3)$$
$$x \to +\infty.$$

Problem (A) is given by the integral operator $A\gamma$, specified in space $L_1(S_p)$, with the values $C^2\Omega$. The necessary properties of operator $A\gamma$ are given in Refs. 5-8.

In the problem under consideration, operator $A\gamma$ has the form /5

$$A\gamma = \frac{1}{4\pi}\int_{S_p} \gamma(p)\frac{d}{d\zeta}\left[\frac{1}{r} + \operatorname{sign} F\frac{1}{r_1}\right]dS; \tag{1,4}$$

where

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2};$$
$$r_1 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta+4\bar{h})^2};$$

$$\operatorname{sign} F = \begin{cases} -1 & \text{- solid screen} \\ +1 & \text{- free surface.} \end{cases}$$

From condition (1.2), we obtain a two-dimensional integral equation of the problem

$$\bar{A}_z\gamma = F(g), \quad g \in S_p. \tag{1,5}$$

The solution will be represented in the form of three components:[6-8]

$$\gamma(g) = \gamma_1(g) + \gamma_2(g) + \gamma_3(g) \qquad g \in S_p,$$

where $\gamma_1$ is a solution of class $C^1(S_p)$, related to the presence of a discontinuity of tangential velocities when surface $S_p$ is crossed;

$\gamma_2$ is a solution of class $C^1(S_p)$ describing inertial motions;

$\gamma_3$ is a singular solution.

An analysis of Eq. (1.5) and a method of setting up the equations for $\gamma_i$ are given in Ref. 6.

We will write the equations for $\gamma_1$ and $\gamma_2$:

$$N_{01}\bar{A}_z\gamma_1 = F_1' + C, \quad g \in S_p; \tag{1,6}$$
$$N_{01}\bar{A}_z\gamma_2' = -ikF_1, \quad g \in S_p. \tag{1,7}$$

Here $F_1 = -\frac{d}{dx}F_2(g)$; $F_2(g)$ is a function describing the shape of the surface S; $N_{01} = -\int^{x} d\tau$.

Constant C is obtained from the solvability condition of Eq. (1.6) in space $C^1(S_p)$.[6]

Performing the calculations, we have for $N_{01}\bar{A}_z\gamma$ [10]

$$N_{01}\bar{A}_z\gamma = \frac{1}{2\pi}\int_{-1}^{+1}\int_{-1}^{+1}\bar{\gamma}(p)\left\{\frac{1}{dy}\frac{1}{(y-\eta)}\frac{\sqrt{(x-\xi)^2+\lambda^2(\eta)(y-\eta)^2}}{(x-\xi)} - \right.$$
$$\left. -\operatorname{sign} F\frac{(x-\xi)[(y-\eta)^2\lambda^2(\eta)(\bar{r}_1^2-\lambda^2(\eta)16\bar{h}^2(\eta)] - \lambda^2(\eta)16\bar{h}^2(\eta)\cdot(\bar{r}_1^2+\lambda^2(\eta)16\bar{h}^2(\eta))]}{\bar{r}_1^2}\right\}dS, \tag{1,8}$$

where

$$\bar{\gamma}(p) = \frac{\gamma(p)}{2\lambda(\gamma_l)}.$$

The singular solution is determined from the equation

$$N\bar{A}_z\gamma = F_1(g); \quad h \in S_p;$$
$$N = -e^{ikx}\int_{\infty}^{x} e^{-ik\tau}d\tau, \qquad (1,9)$$

where k is the chordwise Strouhal number.

Equation (1.9) is obtained by mapping Eq. (1.5) into the space of the velocity potential. /6

2. Let us now consider the Prandtl problem for a wing in an unsteady flow. For a wing of high aspect ratio, the operator $N_{01}\bar{A}_2\gamma$ is approximated by the operator

$$N_{01}\bar{A}_z\gamma = \frac{\lambda(y)}{\pi}\int_{-1}^{+1}\bar{\gamma}(\xi;y)\left[\frac{1}{(x-\xi)} + \operatorname{sign} F\,\frac{(x-\xi)}{(x-\xi)^2+16h^2(y)}\right]dy. \qquad (2,1)$$

The form of singular solution $\gamma_3$ is known for plane flow, and for a wing of high aspect ratio we will take $\gamma_3$ in the form of a solution to the two-dimensional problem, but with a constant dependent on the y coordinate:

$$\gamma_3 = a(y)\sqrt{\frac{1+S}{1-S}}.$$

If

$$\gamma_{12}^0 \in C^1(S) \text{ и } \gamma_2 = -ik\int_{-1}^{x}\gamma_j d\tau; \quad g \in S_p,$$

we have[6]

$$N\bar{A}_z\gamma_{12}^0 = N_{01}\bar{A}_z\gamma_1; \quad g \in S_p. \qquad (2,2)$$

Considering Eqs. (1.6), (1.7), (1.9) and the property (2.2), we obtain

$$N\bar{A}_z\gamma_3 = -C - N_1\bar{A}_z\gamma_{22}.$$

Equation (1.6) can be written in the form

$$N_0\bar{A}_z\gamma_1 = F_1 + C^1, \qquad (2,3)$$
$$N_0 = -\int_{\infty}^{x} d\tau.$$

For $\lambda \to \infty\, C^1 = C$, and for higher aspect ratios, we will take the approximation

$$C = -\frac{1}{\pi}\int_{-1}^{+1}\frac{[F_1-(N_0-N_{01})\bar{A}_z\gamma_1]}{\sqrt{1-x^2}}dx. \qquad (2,4)$$

Then, introducing the operator $p^{-1}$:

$$p^{-1}N_{01}\bar{A}_z\gamma_1 = \int_{-1}^{+1}\gamma_1(\xi)d\xi$$

and taking approximation (2.1) from (2.3), we obtain the Prandtl equation for a regular solution having the same form as the Prandtl equation of the stationary problem:[4]

$$\Gamma_1(y) = \frac{\psi\pi}{\lambda(y)}\left\{\alpha_1 - \frac{1}{2\pi}\int_{-1}^{+1} \Gamma_1'(\eta)\left[\frac{1}{y-\eta} - \frac{y-\eta}{(y-\eta)^2+16\bar{h}^2}\right] d\right\} \qquad (2,5)$$

where

$$\Gamma_1 = \int_{-1}^{+1} \gamma_1(\xi)\, d\xi.$$

The function $\gamma_{22}$ is obtained from the equation

$$N_{01}\bar{A}_z\gamma_{22}' = ik\,C(y). \qquad (2,6)$$

We will introduce a series of assumptions typical of the Prandtl problem:[4,8] /7

$$N_1\bar{A}_z\gamma \approx N_{10}\bar{A}_z\gamma + N_{1\lambda}\bar{A}_z\gamma;$$

$$\gamma(p) = \gamma_1(\xi)\gamma_2(y);$$

$$N_{10}\bar{A}_z\gamma = 2\gamma_2(y)\lambda(y)N_{10}\bar{\bar{A}}_z\gamma_1(\xi);$$

$$N_{1\lambda} = -e^{ikx}\int_{\infty}^{\xi} e^{-ik\tau}\,d\tau;$$

$$N_{10}\bar{\bar{A}}_z\gamma = \frac{1}{2\pi}\left\{\int_{-1}^{+1}\gamma(\xi)\left[\frac{1}{(x-\xi)} + \operatorname{sign} F\frac{(x-\xi)}{(x-\xi)^2+16\bar{h}^2(y)}\right]d\xi + \right.$$

$$\left. + ike^{ikx}\int_{-1}^{+1}\gamma(\xi)\int_{\infty}^{x} e^{-ik\tau}\left[\frac{1}{(\tau-\xi)} + \operatorname{sign} F\frac{(\tau-\xi)}{(\tau-\xi)^2+16\bar{h}^2(y)}\right]d\tau\, d\xi\right\},$$

then

$$a(y)\,2\lambda(y)\,N_{10}\bar{\bar{A}}_z\gamma_1(\xi) = -C(y) - 2\lambda(y)\gamma_{222}(y)N_{10}\bar{\bar{A}}_z\gamma_{221} -$$

$$-N_{1\lambda}'\bar{A}_z\left[a(y)\sqrt{\frac{1+S}{1-S}} + \gamma_{22}\right]; \quad g \in S_\rho. \qquad (2,7)$$

For a wing in an unbounded fluid, both parts of Eq. (2.7) contain the factors $e^{ikx}$. Multiplying Eq. (2.7) by $e^{-ikx}$, we eliminate the variable x.[5,8] This property also applies in the general case, but the equation cannot be solved for $e^{ikx}$ in explicit form, since one can calculate the integrals in $N_{10}\bar{\bar{A}}_z\gamma$ in closed form. We therefore introduce the averaging of Eq. (2.7) in accordance with the rule

$$\bar{B} = \frac{\int_{-1}^{+1} B\sqrt{\frac{1-x}{1+x}}\,dx}{\int_{-1}^{+1} e^{ikx}\sqrt{\frac{1-x}{1+x}}\,dx}.$$

For $\overline{N_{10}\bar{\bar{A}}_z\gamma_1}$ and $\overline{\gamma_{222}'(y)N_{10}\bar{\bar{A}}_z\gamma_{221}}$ we will use

$$\overline{N_{10}\bar{\bar{A}}_z\gamma_1} = \frac{\overline{N_1(k)}}{\psi_1(k,\bar{h})\,2\pi}; \qquad (2,8)$$

$$\overline{\gamma_{222}(y)\, N_{10}\bar{\bar{A}}_z\gamma_{221}} = \frac{\gamma_{222}(y)}{2\pi\psi_2(k;\,\bar h)\,2\lambda(y)\,\lambda^0_{222}(y)}\left[\frac{ik}{\pi}\bar N_2(k)\,C(y) + C_2(y)\,\bar N_3(k)\right]. \quad (2.9)$$

For $\bar h \to \infty$, the representations (2.8) and (2.9) change into the expressions obtained in Ref. 9. The functions $\bar N_i(k)$ were also obtained in Ref. 9.

Then

$$a(y)\,2\lambda(y)\frac{\bar N_1(k)}{\psi_1(k;\,\bar h)\,2\pi} + \frac{\gamma_{222}(y)}{2\pi\psi_2(k;\,\bar h)\,\gamma^0_{222}(y)}\left[\frac{ik}{\pi}\bar N_2(k)\,C(y) + C_2(y)\,\bar N_3(k)\right] =$$
$$= -C(y) - N_{1\lambda}\bar A_z\left[a(y)\sqrt{\frac{1+S}{1-S}} + \gamma_{22}\right]. \quad (2.10)$$

Upon introduction of the new variable $a_\lambda(y) = \int_{-1}^{+1}\left[a(y)\sqrt{\frac{1+S}{1-S}} + \gamma_{221}(y;\,S)\right]dy$ , /8

Eq. (2.8) becomes

$$a_\lambda(y) = \frac{\pi\psi_1(k;\,\bar h)}{2\lambda(y)}\left\{\alpha_e + \frac{\pi L a'_\lambda(\eta)}{N_1(k)}\right\}. \quad (2.11)$$

Here

$$\alpha_e = -\frac{\pi\left\{C(y) + \dfrac{\gamma_{222}(y)}{\psi_2(k;\,\bar h)\,\gamma^0_{222}(y)}\left[\dfrac{ik}{\pi}\bar N_2(k)\,C(y) + C_2(y)\,\bar N_3(k)\right]\right\}}{\bar N_1(k)} +$$
$$+ \frac{\lambda(y)}{\psi_1(k)\,\pi}\gamma_{222}(y)\int_{-1}^{+1}\gamma_{221}(\xi)\,d\xi.$$

As follows from the solution of the two-dimensional problem of unsteady motion of a wing near a screen,[7,8] the function $\gamma(S)$ may be written as

$$\gamma(S) = \psi_s a_0(y)\sqrt{\frac{1+S}{1-S}} + \psi\gamma_1^0(S) + \psi_p\gamma_2^0(S).$$

The functions $\gamma_i^0(S)$, corresponding to the two-dimensional problem for an unbounded fluid, were calculated in Ref. 9. Using these results, we have

$$a_e = \frac{a_0(y)}{2} - \frac{\pi}{N_1(k)}\left[\frac{ik}{\pi}\bar N_2(k)\,C(y) + C_2(y)\,\bar N_3(k)\right]\left(\frac{\psi_p}{\psi_2} - 1\right) + ikC_3(y)\frac{\psi_p}{\psi_1}, \quad (2.12)$$

$La'_\lambda$ being an integral operator whose kernel was obtained in the monograph of Ref. 4.

We must now establish the relations between the functions excluding $\psi_1$ and $\psi_2$ from Eq. (2.9). By definition, when $\lambda \to \infty$

$$a_\lambda(y) = \pi\left[\frac{\psi_s a_0(y)}{2\lambda(y)} + \frac{\psi_p ik\,C_3(y)}{\lambda(y)}\right].$$

The constants $C_i(y)$ are

$$C_2(y) = \frac{2}{\pi}\int_{-1}^{+1} \frac{F_1(x)}{\sqrt{1-x^2}}\,dx = \frac{2}{\pi}\int_{-1}^{+1} \frac{[F_1 - (N_0 - N_{01})\bar{A}_2\gamma_1]}{\sqrt{1-x^2}}\,dx;$$

$$C_3(y) = -\frac{1}{\pi}\int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}}\,F_1(x)\,dx.$$

From Eq. (2.9) we find

$$a_\lambda(y) = \frac{\pi\psi_1(k)}{\lambda(y)}\,\alpha_e^1;$$

$$\alpha_e^1 = \alpha_{e\,\lambda\to\infty} = \frac{a_0(y)}{2} + \frac{\psi_\rho}{\psi_s}\,ik\,C_3(y) -$$

$$-\frac{\pi}{\bar{N}_1(k)}\left[\frac{ik}{\pi}\,\bar{N}_2(k)\,C(y) + C_3(y)\,\bar{N}_3(k)\right]\left(\frac{\psi_\rho}{\psi_s} - 1\right).$$

Comparing the two expressions for $a_\lambda(y)$, we obtain the desired relation

$$\psi_\rho = \psi_2;\quad \psi_1 = \psi_s.$$

Combining the results obtained from Eq. (2.9), we write the integrodifferential equation of a high-aspect-ratio wing near a screen in an unsteady flow /9

$$a_\lambda(y) = \frac{\pi\psi_s}{\lambda(y)}\left[\alpha_e - \frac{1}{2\pi\,|\bar{N}_1(k)|}\int_{-1}^{+1} a_\lambda'(\eta)\,K\left[k_\lambda(y-\eta)\right]d\eta, \qquad (2,13)\right.$$

where

$$\alpha_e = \frac{a_0(y)}{2} + \frac{\psi_\rho}{\psi_s}\,ik\,C_3(y).$$

The kernel K (9) is[4]

$$K(g) = \frac{N_{-1}P\,|y-\eta|}{(y-\eta)} + \frac{y-\eta}{(y-\eta)^2 + 16\bar{h}^2}\,N_{-1}\left(P\sqrt{(y-\eta)^2 + 16\bar{h}^2}\right) -$$

$$-\frac{4\bar{h}}{(y-\eta)^2 + 16\bar{h}^2}\,N_0\left(P\sqrt{(y-\eta)^2 + 16\bar{h}^2}\right).$$

When $K \to 0$, $N_1(k) \to -1$, and from Eq. (2.11) we obtain the Prandtl equation for a wing near a screen in a steady flow.[4] Then

$$K(y-\eta) = \frac{1}{(y-\eta)} + \operatorname{sign} F\,\frac{(y-\eta)}{(y-\eta)^2 + 16\bar{h}^2}.$$

Thus, the Prandtl problem for a wing in an unsteady flow has been reduced to two one-dimensional integral equations (2.5) and (2.13), with Eq. (2.5) corresponding to the equation of the stationary theory.

This result, which is a direct consequence of the separation of the general solution into three components, is of major importance, since by introducing "reduced circulation" type integral quantities, the authors of the known theories [1,2] find a single one-dimensional equation.

3. In determining the aerodynamic coefficients $C_y$ and $C_x$, a close first approximation is given by the solutions given in Refs. 3 and 4.

$$\Gamma(y) = \Phi\sqrt{1-\bar{y}^2}.$$

We will use this representation for an approximate solution of (2.13). Using $a_\lambda(y) = \Phi\sqrt{1-y^2}$ we obtain for constant $\Phi$

$$\Phi = \frac{\dfrac{\pi\psi_s a_e}{\lambda(0)}}{1+\dfrac{\pi\psi_s}{2\lambda(0)\,|\bar{N}_1(k)|}}\zeta k_\lambda, \qquad (3,1)$$

where $\zeta k_\lambda$ is given by the formula from Ref. 4:

$$\zeta k_\lambda = 1-\frac{2}{\pi^2}\int_{-1}^{+1}\sqrt{1-y^2}\int_{-1}^{+1}\frac{\eta}{\sqrt{1-\eta^2}}\left\{K[k_\lambda(y-\eta)]-\frac{1}{(y-\eta)}\right\}d\eta, \qquad (3,2)$$

$k_\lambda$ being the spanwise Strouhal number.

The wing lift coefficient is calculated from the formula

$$C_y = \frac{\lambda}{\pi}\int_{-1}^{+1}\int_{-1}^{+1}\gamma(\xi,\ \eta)\,d\xi\,d\eta. \qquad (3,3)$$

We will write the wing lift coefficient in the form of two components: /10

$$C_y = C_{yv} + C_{yi},$$

where $C_{yv}$ is the lift coefficient determined by the vortex motion;

$C_{yi}$ is the lift coefficient determined by the apparent wing mass.

We find the components $C_{yv}$ and $C_{yi}$:

$$C_{yv} = \frac{a_\infty\psi_s}{1+\dfrac{a_\infty\psi_s}{\pi\lambda\,|\bar{N}_1(k)|}\zeta_{k_\lambda}}\frac{C(k)}{\pi}\int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}F_e^0(x)\,dx+$$

$$+2\psi_p ik\int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}F_e(x)\,dx\left[1-\frac{1}{1+\dfrac{a_\infty\psi_s}{\pi\lambda\,|\bar{N}_1(k)|}\zeta_{k_\lambda}}\right]+$$

$$+2\psi\int_{-1}^{+1}\frac{F_c(x)}{\sqrt{1-x^2}}x\,dx\left\{1-\frac{\psi_s}{\psi\left[1+\dfrac{a_\infty\psi_s}{\pi\lambda\,|\bar{N}_1(k)|}\zeta_{k_\lambda}\right]}\right\}; \qquad (3,4)$$

$$C_{yi} = -\psi_p 2ik\int_{-1}^{+1}\sqrt{1-x^2}F_1(x)\,dx. \qquad (3,5)$$

Here

$$F_e(x) = F_1(x) - [N_0 - N_{01}]\bar{A}_z\gamma_1,$$

$C(k)$ being the Theodorsen coefficient.[2]

In the calculations, the theoretical value $a_\infty = 2\pi$ (allowing for the influence of the viscosity of the fluid) may be taken as $a_\infty \approx 5.45$.

For steady flow $C_{yi} = 0$, and

$$C_{yv} = -\frac{a_\infty \psi}{1+\frac{a_\infty \psi}{\pi\lambda}} \int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}} F_e(x)\,dx +$$

$$+ 2\psi \int_{-1}^{+1} \frac{F_e(x)}{\sqrt{1-x^2}} x\,dx \left[1 - \frac{1}{1+\frac{a_\infty \psi}{\pi\lambda}\zeta}\right]. \qquad (3,6)$$

The first term in Eq. (3.6) gives a known value of the wing coefficient in a bounded flow, and the second term, of order $\frac{1}{\lambda}$, is the refinement introduced into the Prandtl theory by the present method. For a plane lifting surface, the second term in Eq. (3.6) is zero, and the results will fully correspond to the classical theory of a lifting line.[3,4]

The functions $\psi_s$, $\psi_p$, $\psi$ were obtained in Refs. 7, 8, and 11 for translational and rotational wing oscillations.

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